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ABSTRACT

This is the final test for students in the twelfth-grade course Mathematics 30, offered in schools in Alberta, Canada. Intended for administration in June 1985, it contains 52 multiple-choice questions and five written-response problems, with two-and-one-half hours allowed for completion. Approved calculators may be used. Scoring is done by the province education department. (MNS)

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DIPLOMA

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GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Five written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example	**Answer Sheet**
This examination is for the subject area of	A B C D
A. Chemistry	① ② ③ ●
B. Biology	
C. Physics	
D. Mathematics	

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JUNE 1985

PART A

INSTRUCTIONS

There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER.

1. The secant of the angle between the x-axis and the radius that intersects the unit circle at the point $\left(-\frac{12}{13}, -\frac{5}{13} \right)$ is

   A. $-\frac{5}{12}$

   B. $-\frac{12}{5}$

   C. $-\frac{13}{5}$

   D. $-\frac{13}{12}$

2. The function $y = \tan x$, where x is in radians, is

   A. undefined when $x = n\pi$ for each integer n

   B. undefined when $x = -1$ or 1

   C. defined for all values of x

   D. undefined when $x = \frac{\pi}{2} + n\pi$ for each integer n

3. The solution set of $2 \sin^2 \theta = -\sin \theta, 0 \leq \theta < 2\pi$ is

   A. $\left\{ 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

   B. $\left\{ 0, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

   C. $\left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

   D. $\left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

4. If $\sin\theta = -\frac{5}{13}$ and $\cos\theta$ is positive, then $\cot\theta$ is

A. $-\frac{12}{5}$

B. $\frac{5}{12}$

C. $-\frac{13}{12}$

D. $\frac{12}{13}$

5. If $\sin A = \frac{1}{2\sqrt{2}}$ and $\cos B = \frac{1}{\sqrt{2}}$, then the value of $\sec^2 B - \csc^2 A$ is

A. -4
B. -6
C. 2
D. 8

6. The EXACT value of $\cos 105°$ is

A. $\frac{\sqrt{6} - \sqrt{2}}{4}$

B. $\frac{\sqrt{6} + \sqrt{2}}{4}$

C. $\frac{\sqrt{2} - \sqrt{6}}{4}$

D. $\frac{1 - \sqrt{3}}{8}$

7. $\cos\left(\theta - \frac{\pi}{2}\right)$ is equal to

A. $\sin\theta$

B. $-\sin\theta$

C. $\cos\theta$

D. $-\cos\theta$

8. An angle of one radian is defined as

   A. an angle given by the product of the length of an arc of a circle and its radius

   B. an angle that, when it is a central angle of a circle, intercepts an arc of the circle equal to the radius

   C. an angle formed by the ratio of the radius of a circle to the arc subtended by the central angle

   D. an angle that, when it is a central angle of a circle, equals the measure of the subtended arc

9. The measure of 260° expressed in radians is

   A. $\frac{13\pi}{9}$

   B. $\frac{9\pi}{13}$

   C. $\frac{9}{13\pi}$

   D. $\frac{13}{9\pi}$

10. The EXACT value of csc 315° is

   A. $-\frac{\sqrt{2}}{2}$

   B. $\frac{\sqrt{2}}{2}$

   C. $-\sqrt{2}$

   D. $\sqrt{2}$

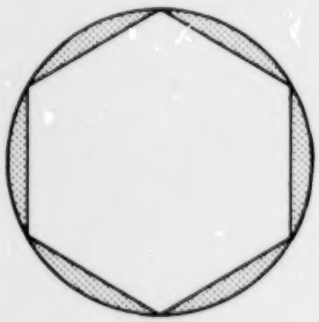
11. In $\triangle ABC$, $\angle A = 120°$, $b = 4$ cm, and $c = 5$ cm. The length of the third side of the triangle is

   A. 8.6 cm
   B. 7.8 cm
   C. 4.6 cm
   D. 2.6 cm

12. A power pole leans 12° from the vertical away from the sun. Its shadow, on level ground, is 23 m long when the angle of elevation of the sun is 15°. The length of the pole is

A. 13.1 m
B. 11.4 m
C. 6.0 m
D. 5.2 m

13. A regular hexagonal picture is set in a circular frame of radius 15 cm as shown in the figure. To the nearest cm^2, the area of the frame not covered by the picture (i.e., the shaded area) is



A. 97 cm^2

B. 122 cm^2

C. 585 cm^2

D. 706 cm^2

14. The set of values of F for which $x^2 + y^2 - 2x + 4y + F = 0$ defines a circle with radius greater than zero is

A. $F < 5$
B. $F > 5$
C. $F < 20$
D. $F > -20$

15. The equation of a circle is $x^2 + y^2 + 4x - 10y - 20 = 0$. The centre and radius respectively of this circle are

A. $(2, -5)$, 49
B. $(-2, 5)$, 49
C. $(2, -5)$, 7
D. $(-2, 5)$, 7

16. A parabola whose equation is $x^2 + 10y = 0$ has its focus at

A. $\left(0, \frac{5}{2}\right)$

B. $\left(0, -\frac{5}{2}\right)$

C. $\left(-\frac{5}{2}, 0\right)$

D. $\left(\frac{5}{2}, 0\right)$

17. What is the equation of a parabola for which the vertex is $(1, -2)$, the axis is parallel to the x-axis, and the directrix is the line $x - 3 = 0$?

A. $(y + 2)^2 = -8x + 8$

B. $(y + 2)^2 = 8x - 8$

C. $(x - 2)^2 = -8y + 8$

D. $(x - 2)^2 = 9y + 9$

18. The support of a bridge is built in the form of a parabolic arch. Its width at the base is 80 m, and its height is 20 m. If the vertex of the parabola is at the origin, then the defining equation is

A. $x^2 = -80y$

B. $x^2 = -40y$

C. $x^2 = -10y$

D. $x^2 = -320y$

19. One focus of the ellipse defined by $64x^2 + 100y^2 = 6400$ is at

A. $(-2, 0)$
B. $(-6, 0)$
C. $(8, 0)$
D. $(10, 0)$

20. One focus of an ellipse is at $(-5, 0)$ and one vertex is at $(-10, 0)$. The equation of the ellipse centred at the origin is

A. $\frac{x^2}{10} + \frac{y^2}{5} = 1$

B. $\frac{x^2}{20} + \frac{y^2}{10} = 1$

C. $\frac{x^2}{100} + \frac{y^2}{75} = 1$

D. $\frac{x^2}{100} + \frac{y^2}{25} = 1$

21. An elliptical mirror with major axis 10 cm is placed in standard position. A beam of light passes through one focus of the mirror at (3, 0) and strikes the mirror at point A (4, y). The value of y is closest to

A. 6.0
B. 3.0
C. 2.4
D. 2.0

22. The equation of the hyperbola that is centred at the origin and has a horizontal transverse axis is

A. $4x^2 + 25y^2 = 100$

B. $4y^2 + 25x^2 = -100$

C. $4x^2 - 25y^2 = 100$

D. $25y^2 - 4x^2 = 100$

23. The length of the conjugate axis of the hyperbola $3x^2 - 7y^2 = 21$ is

A. 14

B. 6

C. $2\sqrt{7}$

D. $2\sqrt{3}$

24. The path of a meteor is hyperbolic, and may be represented graphically by a point moving so that the difference of its distances from $(0, 20)$ and $(0, -20)$ is always 30 units. If the meteor strikes an object located at $(5, y)$, the positive value of y will be closest to

A. 14.0
B. 16.0
C. 16.5
D. 30.5

25. The number of positive integers between 30 and 435 that are divisible by 8 is

A. 50
B. 51
C. 52
D. 53

26. The sum of n terms of an arithmetic sequence is 570. The common difference is 5 and the third term is 30. An expression that determines the number of terms is

A. $n^2 + 9n + 228 = 0$

B. $n^2 + 9n - 228 = 0$

C. $n^2 + 7n + 228 = 0$

D. $n^2 + 7n - 228 = 0$

27. The geometric series with common ratio $\sqrt{2}$ is

A. $3\sqrt{2} + 4\sqrt{2} + 5\sqrt{2} + \cdots$

B. $3\sqrt{2} + 6 + 6\sqrt{2} + \cdots$

C. $3\sqrt{2} + 6\sqrt{2} + 12\sqrt{2} + \cdots$

D. $\sum_{n=1}^{4} (3 + n\sqrt{2})$

28. The sum of n terms in a geometric series is 728. If the first term is 2 and the common ratio is 3, then the number of terms in the series is

A. 5
B. 6
C. 7
D. 8

29. Using sigma notation, the series $4 + 9 + 16 + 25$ is

A. $\sum_{k=1}^{4} (3k + 1)$

B. $\sum_{k=1}^{4} (k^2 + 3)$

C. $\sum_{k=2}^{5} k^2$

D. $\sum_{k=2}^{5} 2^k$

30. $\lim_{n \to \infty} \left(\frac{6n^2}{2n^3 + n + 3} - \frac{5n}{10n + 5} \right)$ equals

A. -5

B. -2

C. $-\frac{1}{2}$

D. $2\frac{1}{2}$

31. Which of the following is a convergent sequence?

A. $3, 9, 27, 81, \cdots, 3^n, \cdots$

B. $10, 8, 6, 4, \cdots, (12 - 2n), \cdots$

C. $4, -4, 4, -4, \cdots, -4(-1)^n, \cdots$

D. $5, 1, \frac{1}{5}, \frac{1}{25}, \cdots, \frac{25}{5^n}, \cdots$

32. The limit of the sequence $7, 8\frac{1}{2}, 7\frac{2}{3}, 8\frac{1}{4}, 7\frac{4}{5}, \cdots, 8 + \frac{(-1)^n}{n}, \cdots$ as $n \to \infty$ is

A. non-existent
B. 7
C. 8
D. 9

33. The sum of the geometric series $8 - 4 + 2 - 1 + \cdots$ is

A. $\frac{8}{3}$

B. $\frac{16}{3}$

C. 12

D. 16

34. The range of the data shown at the right is

17	19	24	27	13
16	23	27	25	27
13	27	28	26	24
14	27	21	23	19

A. 28
B. 27
C. 20
D. 15

35. The standard deviation (correct to two decimal places) of the population shown at the right is

61	59	62	60	60	58
63	60	61	61	56	59

A. 1.78
B. 1.79
C. 1.80
D. 2.05

36. On a diploma examination, the mean of all the scores was 57 with a standard deviation of 9.3. If Sue's score on the examination was 65, what was her z-score?

A. -1.36
B. 1.36
C. -0.86
D. 0.86

37. A brush manufacturer determines the mean life of his brushes to be 7 years, with a standard deviation of 3 years. If he guarantees his brushes for 1 year, the percentage of brushes that he will have to replace is

A. 66%
B. 48%
C. 34%
D. 2%

38. On a certain cold day in winter the mean waiting time for a taxi was 45 min with a standard deviation of 10 min. Of 200 callers, the number who received a taxi within 30 min was

A. 13
B. 44
C. 67
D. 87

39. The mean time for a race trial is 4 min with a standard deviation of 5 s. Only athletes with times less than 3 min 59 s will be selected for the final. If 20 athletes take the trial, the number expected to reach the final is

A. 2
B. 8
C. 10
D. 12

40. It is assumed that the achievement scores on a certain test are normally distributed with $\sigma = 12$ and $\mu = 62$. The probability to 2 decimal places that a student's score will be less than 40 is

A. 0.15
B. 0.03
C. 0.43
D. 0.47

41. For a normal distribution, the mean (μ) is 12 and the standard deviation (σ) is 3. The probability to 2 decimal places of x for the interval $2 \leq x \leq 10.8$ is

A. 0.28
B. 0.21
C. 0.18
D. 0.09

42. $\dfrac{(4^{n+1})\,(2^{2n-1})\,(16^n)}{32^n}$ equals

A. 2^{2n+2}

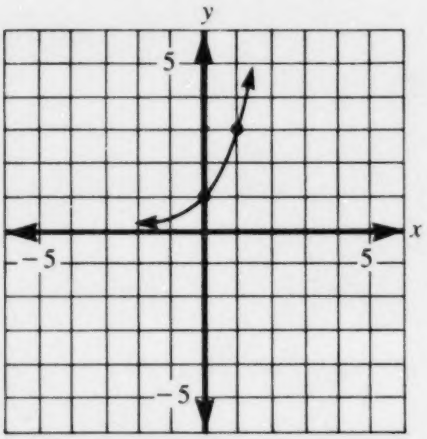
B. 2^{2n+3}

C. 2^{3n+2}

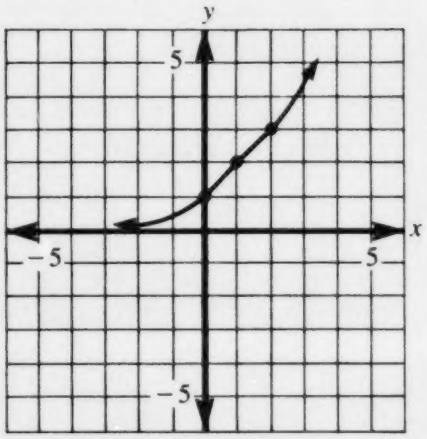
D. 2^{3n+1}

43. The graph of the exponential function $y = 3^x$ is BEST represented by

A.



B.



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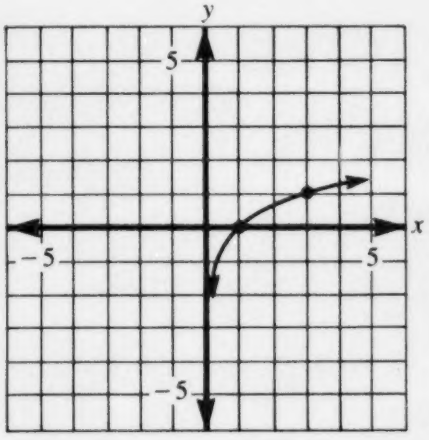

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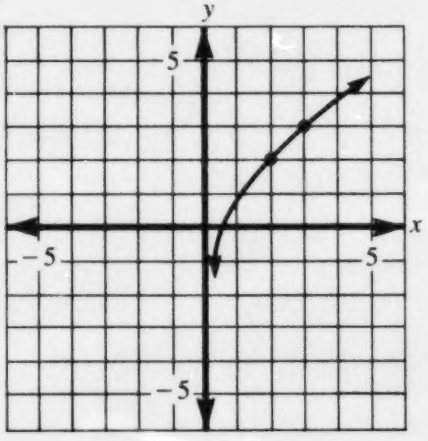


44. The logarithmic form of $49^{\frac{1}{2}} = 7$ is

A. $\log_7\left(\frac{1}{2}\right) = 49$

B. $\log_7(49) = \frac{1}{2}$

C. $\log_{\frac{1}{2}}(7) = 49$

D. $\log_{49}(7) = \frac{1}{2}$

45. If $A = \log(2x), x > 0$ and $B = \log(3x), x > 0$, then the exponential form of $y = B - A$ is

A. $10^y = 1.5$

B. $10^y = x$

C. $10^y = 0$

D. $10^x = y$

46. If $\log_8(x + 5) - \log_8(x - 2) = 1$, then the value of x is

A. -3
B. 1
C. 3
D. 8

Use the information below to answer question 47.

Growth of Bacteria

$N(t) = N_o\left(2^{\frac{t}{40}}\right)$, where $N(t)$ = final number of bacteria

N_o = initial number of bacteria

t = time in minutes

47. The logarithmic expression for the time (t) it takes the number of bacteria to increase from 50 000 to 700 000 is

A. $\dfrac{40}{\log_2(14)}$

B. $\dfrac{\log_2(14)}{40}$

C. $40 \log_2(14)$

D. $\log_2\left(2^{\frac{t}{40}}\right)$

48. When $2x^5 + 10x^4 - x^3 - 5x^2 + 4x + 20$ is divided by $x - 5$, the remainder is

A. 0
B. 40
C. 12 290
D. 12 500

49. A factor of $x^3 - x^2 - 10x - 8$ is

A. $x + 4$
B. $x - 1$
C. $x - 2$
D. $x - 4$

50. If the x-intercepts of the graph of a polynomial function are 3, $\sqrt{3}$, and $-\sqrt{3}$, then the polynomial could be

A. $x^3 + 3x^2 + 3x + 9$

B. $x^3 + 3x^2 - 3x - 9$

C. $x^3 - 3x^2 - 3x + 9$

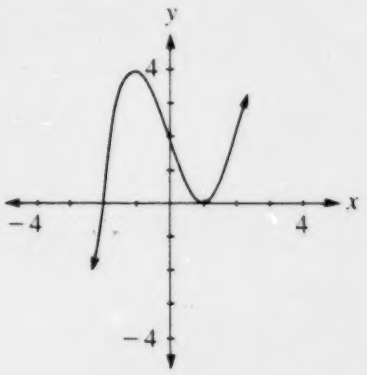
D. $x^3 - 3\sqrt{3}x + 3\sqrt{3}$

51. If -1 and -2 are x-intercepts of the graph of $y = x^3 + ax^2 - x + b$, then the values of a and b respectively are

A. 2, 2
B. 2, -2
C. -2, 2
D. -2, -2

52. If $P(x) = (x - 1)^2(x + 2)$, then the graph of $P(x)$ is

A.



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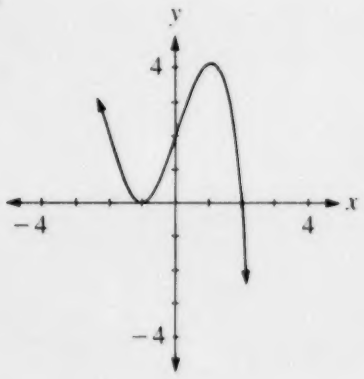

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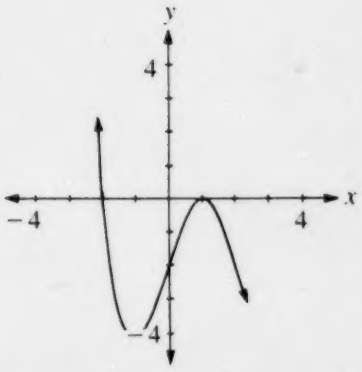

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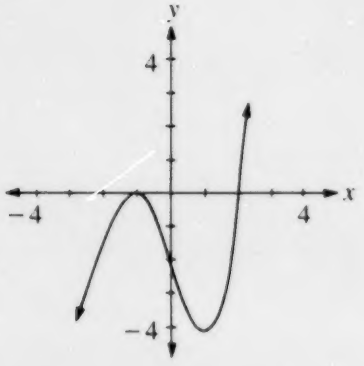


YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

Please write your answers in the examination booklet as neatly as possible.

Show all pertinent calculations and formulas, and give your answers to the correct number of significant figures.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

TOTAL MARKS: 13

START PART B IMMEDIATELY

(3 marks) 1. What is the area of a triangular plot that has sides of 20 m, 25 m, and 30 m?

(3 marks) 2. Determine the equation of the circle passing through the points $(5, 1)$, $(3, 3)$, and $(-1, -5)$.

(3 marks) **3.** At the end of every six months $500 is deposited into a fund that earns interest at 8% per annum compounded quarterly. What is the total amount in the account immediately after the 7th deposit is made?

(2 marks) 4. An IQ test was given to the employees of a computer firm. The results were normally distributed with a mean of 110 and a standard deviation of 15. If 2000 employees took the test, how many scored between 115 and 135?

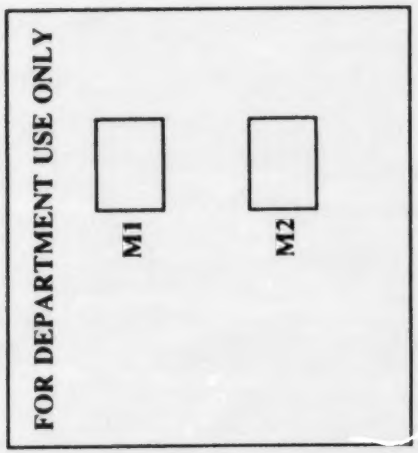
(2 marks) 5. Give the complete factorization of $x^4 + 4x^3 - 2x^2 - 12x + 9$. (All steps must be shown.)

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

TEAR-OUT SHEET

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

FOLD AND TEAR ALONG PERFORATION



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MATHEMATICS 30

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MATHEMATICS 30